Computer Graphics III
Spherical integrals, Light & Radiometry – Exercises

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Reminders & org

- **Renderings due next week**
  - Upload to google drive, show on the big screen, 5 minutes per team (how many teams do we have)
- Papers for presentations in the lab – 7.11., 21.11,
  - ACM TOG special issue on production rendering https://dl.acm.org/citation.cfm?id=3243123&picked=prox
- Reminder – choose papers for the exam
  - http://kesen.realtimerendering.com/
- Log your choices here
  - https://docs.google.com/document/d/128e4Dgh0IvH64DI6Ohu2eRGth0m5i8WlKpDwNyJzpVM/edit?usp=sharing
- Decide assignments track vs. individual project track by Wed, Oct 31st 2018.

PEN & PAPER EXERCISES

Surface area of a (subset of a) sphere

- Calculate the surface area of a unit sphere.
- Calculate the surface area of a spherical cap delimited by the angle θ_o measured from the north pole.
- Calculate the surface area of a spherical wedge with angle ϕ_o.

Solid angle

- What is the solid angle under which we observe an (infinite) plane from a point outside of the plane?

- Calculate the solid angle under which we observe a sphere with radius R, the center of which is at the distance D from the observer.

Isotropic point light

- **Q:** What is the emitted power (flux) of an isotropic point light source with intensity that is a constant I in all directions?

Isotropic point light

- **A:** Total flux:

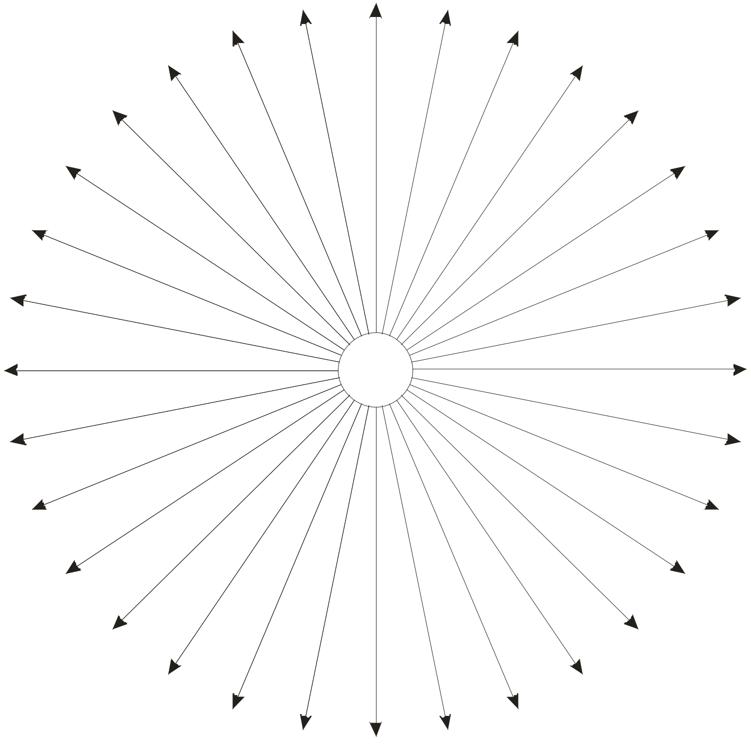
$$\Phi = \int_\Omega I(\omega)\,\mathrm{d}\omega = \left| \begin{array}{c} \mathit{substitute}: \\ \mathrm{d}\omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \end{array} \right|$$

$$= I \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

$$= I\,2\pi \left[-\cos\theta \right]_0^\pi$$

$$= 4\pi I$$

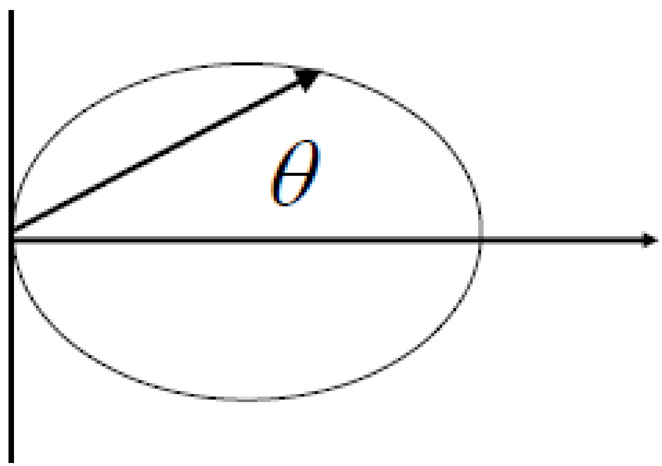
$$I = \frac{\Phi}{4\pi}$$



Cosine spot light

- What is the power (flux) of a point source with radiant intensity given by:

$$I(\omega) = I_0 \max\{0, \omega \cdot \vec{d}\}^s$$



Spotlight with linear angular fall-off

- What is the power (flux) of a point light source with radiant intensity given by:

$$I(\theta, \phi) = \begin{cases} I_0 & \theta \leq \alpha \\ I_0 \dfrac{\beta - \theta}{\beta - \alpha} & \alpha < \theta < \beta \\ 0 & \theta \geq \beta \end{cases}$$

Constant part

$$\Phi_1 = \int_0^{2\pi} \int_0^{\alpha} I_0 \sin\theta d\theta d\phi = I_0 2\pi (1 - \cos\alpha).$$

Linear part

$$\Phi_2 = \int_0^{2\pi} \int_\alpha^{\beta} I_0 \frac{\beta - \theta}{\beta - \alpha} \sin\theta d\theta d\phi = I_0 \frac{2\pi}{\beta - \alpha} \int_\alpha^\beta (\beta - \theta) \sin\theta d\theta \qquad (1)$$

The last integral is the sum of the following two integrals:

$$\int_\alpha^\beta \beta \sin\theta d\theta = \beta\cos\alpha - \beta\cos\beta \qquad (2)$$

$$-\int_\alpha^\beta \theta \sin\theta d\theta = \Big| \sin\theta - \theta\cos\theta \Big|_\beta^\alpha = \sin\alpha - \alpha\cos\alpha - \sin\beta + \beta\cos\beta \qquad (3)$$

Plugging (2) and (3) into (1) and rearranging, we get

$$\Phi_2 = I_0 \frac{2\pi}{\beta - \alpha} [(\beta - \alpha)\cos\alpha + \sin\alpha - \sin\beta] = I_0 2\pi \left[\cos\alpha - \frac{\sin\beta - \sin\alpha}{\beta - \alpha} \right]. \qquad (4)$$

Total flux

$$\boxed{\Phi = \Phi_1 + \Phi_2 = I_0 2\pi \left[1 - \frac{\sin\beta - \sin\alpha}{\beta - \alpha} \right]} \qquad (5)$$

Irradiance due to a Lambertian light source

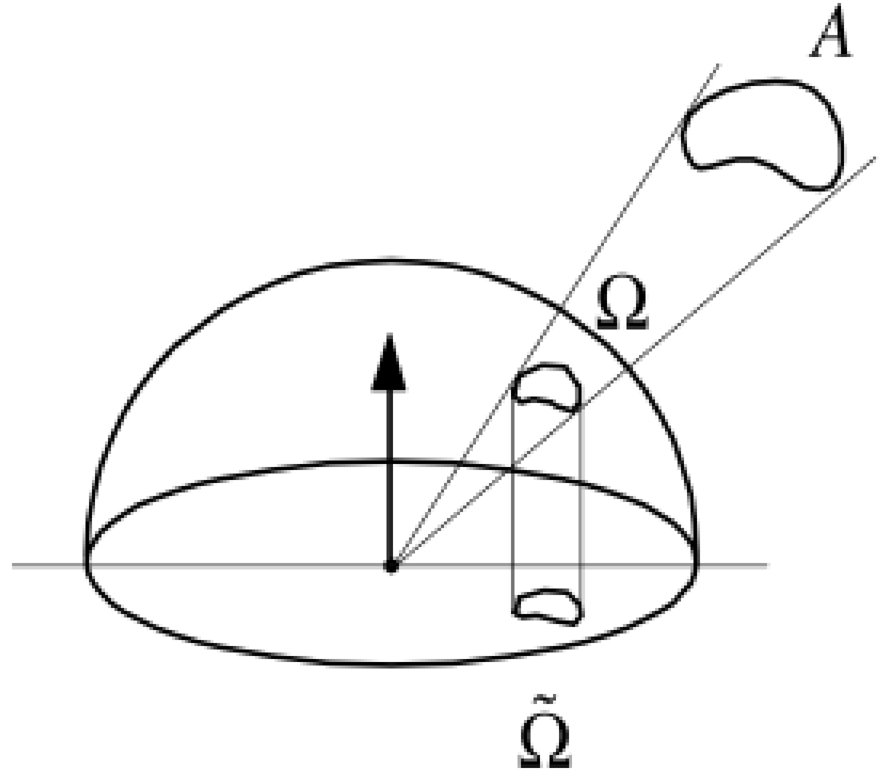
- What is the irradiance $E(\mathbf{x})$ at point $\mathbf{x}$ due to a uniform Lambertian area source observed from point $\mathbf{x}$ under the solid angle Ω?

Uniform Area Source

$$E(x) = \int_{H^2} L \cos\theta \, d\omega$$

$$= L \int_{\Omega} \cos\theta \, d\omega$$

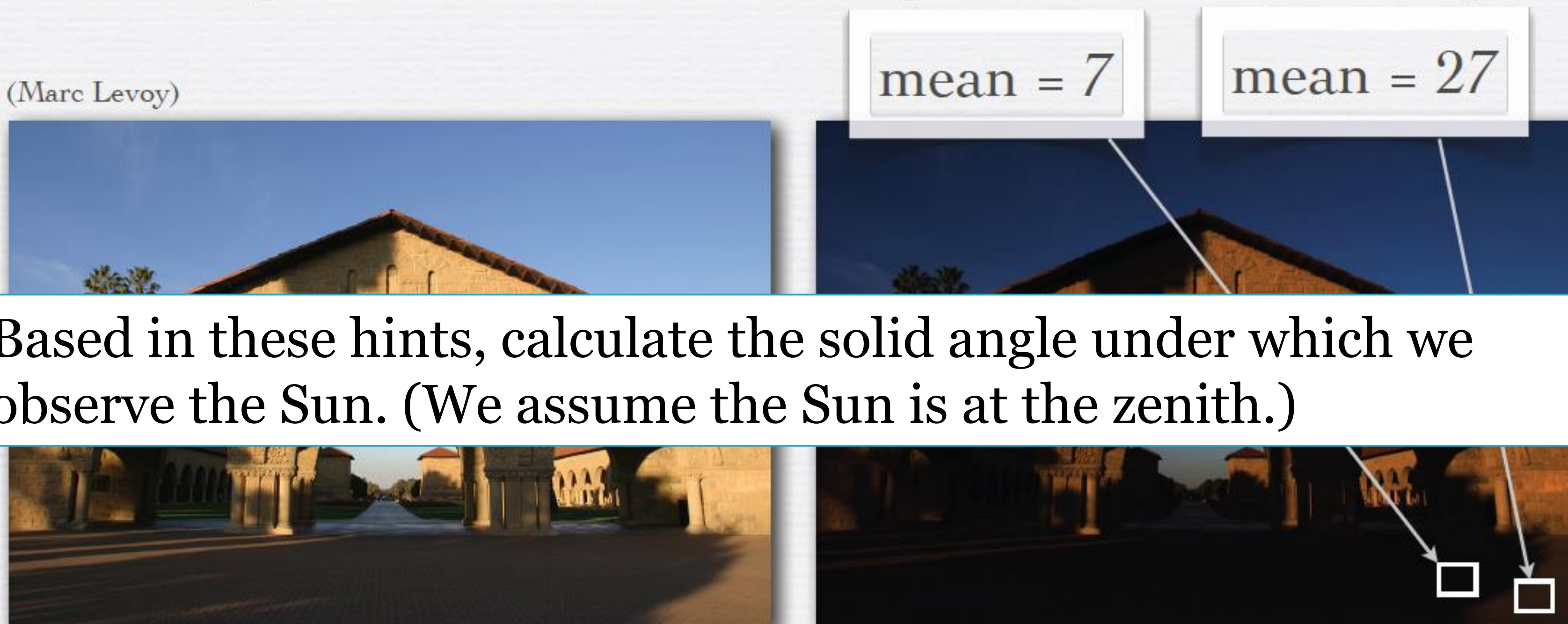
$$= L\tilde{\Omega}$$



How dark are outdoor shadows?

- luminance arriving on a surface from a full (overhead) sun is 300,000 × luminance arriving from the blue sky, but the sun occupies only a small fraction of the sky
- illuminance on a sunny day = 80% from the sun + 20% from blue sky, so shadows are 1/5 as bright as lit areas (2.3 f/stops)

(Marc Levoy)



JPEG file RAW, linearly boosted

Based in these hints, calculate the solid angle under which we observe the Sun. (We assume the Sun is at the zenith.)

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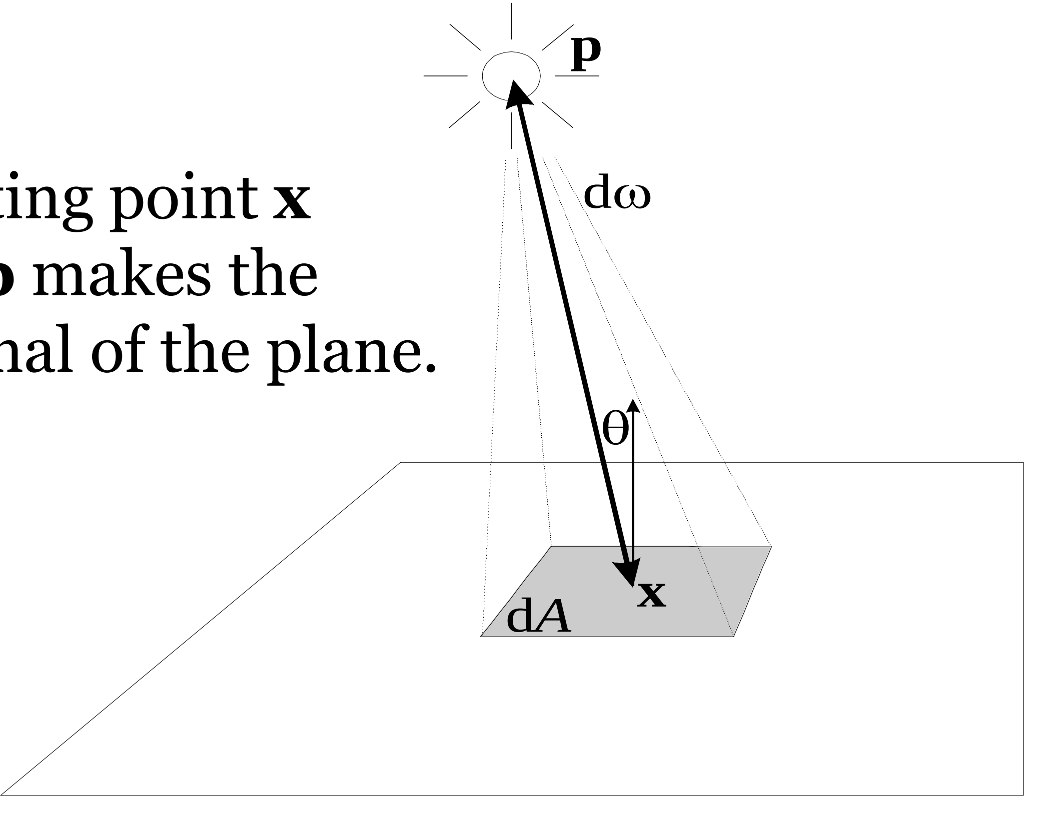
Irradiance due to a point source

- What is the irradiance at point **x** on a plane due to a point source with intensity $I(\omega)$ placed at the height h above the plane.

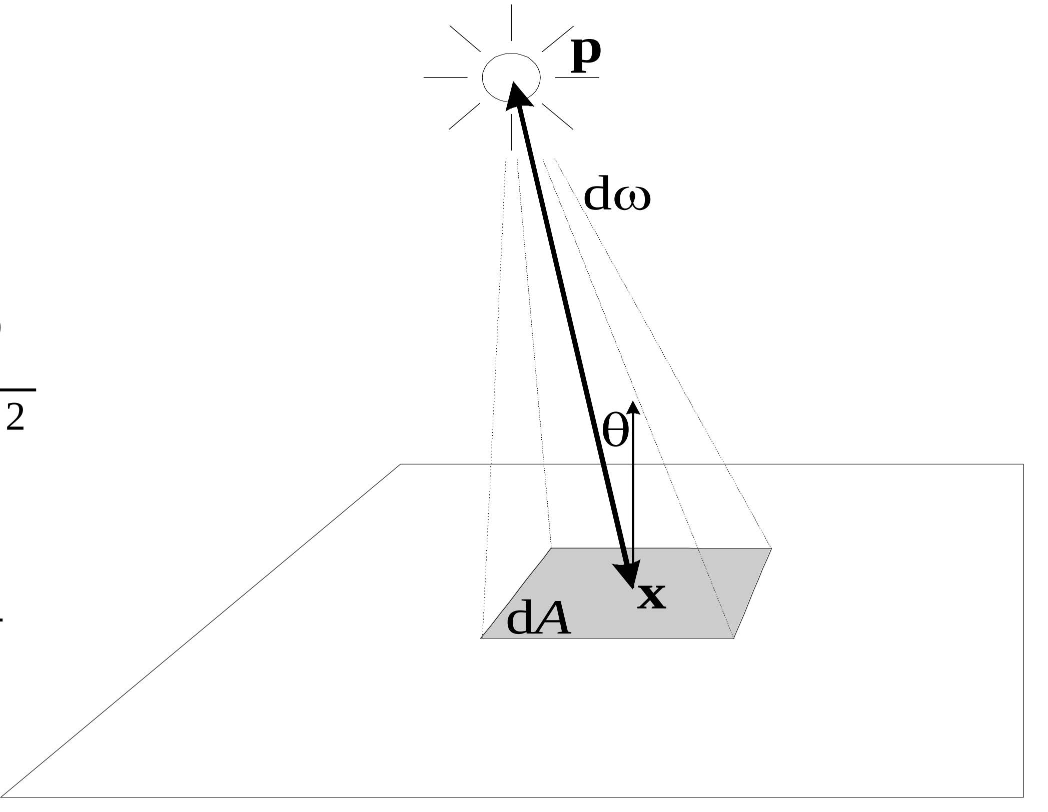
- The segment connecting point **x** to the light position **p** makes the angle θ with the normal of the plane.



Irradiance due to a point source

- Irradiance of a point on a plane lit by a point source:

$$
\begin{aligned}
E(\mathbf{x}) &= \frac{d\Phi(\mathbf{x})}{dA} \\
&= \frac{I(\mathbf{p} \to \mathbf{x})\,d\omega}{dA} \\
&= I(\mathbf{p} \to \mathbf{x}) \frac{\cos\theta}{\|\mathbf{p}-\mathbf{x}\|^2} \\
&= I(\mathbf{p} \to \mathbf{x}) \frac{\cos^3\theta}{h^2}
\end{aligned}
$$



Area light sources

- Emission of an area light source is fully described by the emitted radiance $L_e(\mathbf{x},\omega)$ for all positions on the source $\mathbf{x}$ and all directions ω.

- The total emitted power (flux) is given by an integral of $L_e(\mathbf{x},\omega)$ over the surface of the light source and all directions.

$$\Phi = \int_A \int_{H(\mathbf{x})} L_e(\mathbf{x},\omega) \cos\theta \, \mathrm{d}\omega \, \mathrm{d}A$$

Diffuse (Lambertian) light source

- What is the relationship between the emitted radiant exitance (radiosity) $B_e(\mathbf{x})$ and emitted radiance $L_e(\mathbf{x}, \omega)$ for a Lambertian area light source?

Lambertian source =
emitted radiance does not depend on the direction ω

$$L_e(\mathbf{x}, \omega) = L_e(\mathbf{x}).$$

Diffuse (Lambertian) light source

- $L_e(\mathbf{x}, \omega)$ is constant in ω
- Radiosity: $B_e(\mathbf{x}) = \pi L_e(\mathbf{x})$

$$B_e(\mathbf{x}) = \int_{H(\mathbf{x})} L_e(\mathbf{x}, \omega) \cos\theta \, \mathrm{d}\omega$$
$$= L_e(\mathbf{x}) \int_{H(\mathbf{x})} \cos\theta \, \mathrm{d}\omega$$
$$= \pi L_e(\mathbf{x})$$

Uniform Lambertian light source

- What is the total emitted power (flux) Φ of a **uniform** Lambertian area light source which emits radiance L_e
  - Uniform source – radiance does not depend on the position, $L_e(\mathbf{x}, \omega) = L_e = \text{const.}$

Uniform Lambertian light source

- $L_e(\mathbf{x}, \omega)$ is constant in $\mathbf{x}$ and $\boldsymbol{\omega}$

$$\boldsymbol{\Phi_e = \mathrm{A}\,\mathrm{B}_e = \pi\,\mathrm{A}\,L_e}$$